Mother-Daughter Nuclear Decay

Fluxion Example Description

1 Physical Background

In this simple simulation, a radioactive parent isotope **A** decays into a daughter product **B**, which is also radioactive, and finally decays into a stable third isotope **C**. The relationship between the number of nuclei of each isotope is quickly established.
The rate of change for the number of nuclei of isotope **A** is given by:

$$N'_A = -\lambda_A \cdot N_A \tag{1}$$

Since isotope **A** decays into isotope **B** and this in turn is radioactive, the following applies:

$$N'_B = \lambda_A \cdot N_A - \lambda_B \cdot N_B \tag{2}$$

The third isotope **C** is stable, i.e.

$$N'_C = \lambda_B \cdot N_B \tag{3}$$

2 Simulation

In most cases, the decay constants λ_A and λ_B are not known and also don't provide an intuitive way of understanding the decay. A better (and, above all, more descriptive) measure for the decay of an isotope is the half-life, which indicates after what time T the initial quantity/number of nuclei has been reduced by half. Assuming knowledge about the solution of the decay equation (1), the decay constant can be calculated from the half-life. It is easy to see that the following equation is a solution to (1):

$$N_A(t) = N_{A,0} \cdot exp(-\lambda_B \cdot t) \tag{4}$$

Recalling the definition of half-life, we obtain:

$$N_A(T_A) = \frac{N_{A,0}}{2} \tag{5}$$

Where T_A is the half-life of isotope **A**. Substituting this into our solution shown in (4) we obtain the decay constant:

$$\lambda_A = \frac{ln(2)}{T_A} \tag{6}$$

It follows that this also applies to isotope **B**.