Damped Oscillator

Fluxion Example Description

1 Physical Background

First, we define our usual Newtonian relationships, i.e.

$$x' = v \tag{1}$$

and

$$v' = a \tag{2}$$

Now, in order to calculate the position from the acceleration, we define the formula for the acceleration. A damped harmonic oscillator is described by the following differential equation:

$$F = m \cdot a = -\beta \cdot v - D \cdot x \tag{3}$$

where $\beta \cdot v$ is a frictional force proportional to the velocity and $D \cdot x$ is a restoring force proportional to the displacement from equilibrium.
Usually, Equation 3 is written with the following substitution:

$$a = -2 \cdot \delta \cdot v - \omega_0^2 \cdot x \tag{4}$$

Where

$$\delta = \frac{\beta}{2 \cdot m} \tag{5}$$

is the damping constant, and

$$\omega_0 = \sqrt{\frac{D}{m}} \tag{6}$$

is the angular frequency of the undamped harmonic oscillator (the natural frequency of the system).

Now we can distinguish three cases:

- **Underdamping**: If $\delta < \omega_0$ then the system will oscillate around the equilibrium position until it comes to a rest.

- **Overdamping**: If $\delta > \omega_0$ then the system will approach the zero position without oscillating, but it will be slower than the underdamped case.

- **Critical Damping**: If $\delta = \omega_0$ then the system will return the fastest from its displacement to the equilibrium position.

Which case would be preferable for shock absorbers?